Minor Test 1 (Feb 11, 2016)

Time: 1 hour Max. Marks: 25

1. Consider a discrete time Markov process $\{x(k)\}$ with known transition density function $f[x^k|x^{k-1}]$, for $k = 1, 2, \ldots$. Show that the two-stage transition density function $f[x^k|x^{k-2}]$ may be obtained from the Chapman-Kolmogorov Equation

$$f[x^k|x^{k-2}] = \int_{-\infty}^{\infty} f[x^k|x^{k-1}]f[x^{k-1}|x^{k-2}]dx^{k-1}.$$

(8)

2 (a) Consider a random process $\{x(n)\}$ given as

$$x(n) = Asin(n\omega_0 + \phi)$$

where ω_0 is a constant and A, ϕ are random variables. Show that the autocorrelation sequence of $\{x(n)\}$ is given as

$$r_x(k) = \frac{1}{2}E\{A^2\}cos(k\omega_0)$$

(3)

(b) Extend this result to the case when

$$x(n) = \sum_{l=1}^{L} A_l sin(n\omega_l + \phi_l),$$

where the random variables A_l, ϕ_l are uncorrelated, to show that

$$r_x(k) = \sum_{l=1}^{L} \frac{1}{2}E\{A^2\}cos(k\omega_l).$$

(3)

(c) What can you say about the Power Spectrum of $\{x(n)\}$? (2)

3. Consider an MA(q) process $x(n)$ generated by the all-zero filter with complex-valued coefficients $b_q(k)$

$$H(z) = \sum_{k=0}^{q} b_q(k) z^{-k}$$

(a)Show that the Power Spectrum of $x(n)$ can be written as

$$P_x(z) = \sigma_v^2 B_q(z) B_q^*(1/z^*)$$

(3)

(b)Show also that the autocorrelation values can be written as

$$r_x(k) = \sigma_v^2 b_q(k) * b_q^*(-k) = \sigma_v^2 \sum_{l=0}^{q-|k|} b_q(l+|k|) b_q^*(l)$$

(3)

(c)Write the Yule-walker Equations for the all-~~pole~~ zero model, to solve for the unknown coefficients from the known values of autocorrelation lags, for lags $\{0, 1, 2\}$. Are these equations linear? What does it tell you about finding the parameters of an MA process? Is it simpler than finding the parameters of an AR process or more difficult, in general? (3)